Homework #7 (100 points) - Show all work on the following problems:
(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (20 points): Start with the following scalar and vector potentials:
$V(\vec{r},t) = 0, \vec{A}(\vec{r},t) = -\frac{1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$

1a (10 points): Find the corresponding electric and magnetic fields, and the charge and current distributions that produce them.

1b (10 points): Use the gauge function $\lambda = -\frac{1}{4\pi\varepsilon_0}\frac{qt}{r}$ to transform the potentials to a form which should look much more familiar.

Problem 2 (20 points): Start with the potentials $V(\vec{r},t) = 0, \vec{A}(\vec{r},t) = A_0 \sin(kx - \omega t)\,\hat{y}$, with A_0, k, and ω constants. Find the electric and magnetic fields, and check that they satisfy all four of Maxwell's equations in vacuum. What condition must be satisfied by k and ω? Seem familiar?

Problem 3 (30 points): A point charge $q(t)$ at the origin (i.e. $\rho(\vec{r}) = q(t)\delta^3(\vec{r})$) is steadily increased by a volume current density $\vec{J}(\vec{r}) = -\frac{1}{4\pi r^2}\frac{dq}{dt}\hat{r}$.

3a (10 points): Explicitly verify that the charge continuity equation holds.

3b (10 points): Find the scalar and vector potentials in the Coulomb gauge.

3c (10 points): Find the electric and magnetic fields, and check that they satisfy all four of Maxwell's equations in vacuum.

Problem 4 (30 points): Assume that the loop of wire at right has a current that increases linearly with time (i.e. $I(t) = kt$ for all times t). Calculate the retarded vector potential $\vec{A}$ at the origin. Assume that the loop lies in the x-y plane with the center of curvature of the two arcs at the origin. From $\vec{A}$, find the electric field at the origin. Since you only know the vector potential at one point, you can't use it to find the magnetic field.